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MEASUREMENT OF MECHANICAL VIBRATION DAMPING IN ORTHOTROPIC, COMPOSITE AND ISOTROPIC PLATES BASED ON A CONTINUOUS SYSTEM ANALYSIS

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Abstract—The problem of free and forced transverse vibration of an orthotropic, composite, and isotropic thin square plates with uniformly distributed damping and simply supported boundary conditions has been solved, using a modal expansion technique. A load of the type $P_0 \cos \Omega t$ applied at the center of plate has been considered and the phase angle between the forcing function and the vibration response at the center, as a function of the forcing frequency and the damping parameter determined. This theoretical relationship together with the experimentally measured phase angle between the applied mechanical forcing and the resulting vibration response at various forcing frequencies was used to determine an equivalent viscous damping parameter. This technique has been found to be particularly useful for the measurement and comparison of the relative damping in composite or orthotropic materials. Also, a theoretical relation for the energy loss due to viscous damping in vibrating plates has been developed and the theoretical energy loss at various frequencies has been compared with the experimentally measured energy loss at the same frequencies. Typical damping results are presented for aluminum, steel and aluminum/graphite-fiber composite materials.

1. INTRODUCTION

Mechanical vibration deals with the oscillatory motion of a physical system and is determined by the three parameters; the mass, the stiffness and the damping of the system. Material damping is the dissipation of energy due to periodic variation of the applied stress level. Since the material damping is an important design factor in controlling noise and mechanical vibration, considerable work is being done in this field.

The three most common methods of measuring damping[1] are: (1) Log decrement method; (2) Frequency band width method; (3) Phase angle method. Of the three methods mentioned above, the first two methods are being widely used. The log decrement is usually determined using the rate of decay of either a torsional pendulum or a cantilever beam. In the frequency bandwidth method[2] the driving frequency is changed until the resonant condition is obtained. Then the frequency above and below the resonant frequency, at which the amplitude of vibration is one half that at resonance is determined. If ω_n is the resonant frequency and $\Delta\omega$ the change of frequency from the half amplitude point below resonance to the half amplitude point above resonance, then the log decrement is given as

$$\delta = \frac{\pi \cdot \Delta\omega}{\omega_n \cdot \sqrt{3}} \tag{1.1}$$

Both the methods mentioned above assume a single degree of freedom system.

The phase angle method used up to the present also implicitly assumes a single degree of freedom system and considers the phase lag between the applied forcing and resulting system displacement as a means of damping determination.

Smith and Berns [3] used this approach for an estimation of system damping. They considered a simply supported beam and measured the phase angle between the applied force and the vibration response, by exciting the beam at the center. They have considered the beam as a single degree of freedom system and have reported that the method is suitable for high frequencies (near resonant frequency). However, they have failed to consider the variation of phase angle with the change in frequency and amplitude of vibration.

Pisarenko *et al.*[4] considered a circular plate with dissipation as the only source of energy loss and determined the resonance oscillations of a plate subjected to alternating harmonic loading. Their numerical results show that the log decrement is a function of applied frequency and amplitude, and decreases as the frequency is increased.

In the present work, an attempt has been made to obtain the critical damping ratio at small amplitudes of vibration of plate structures as a function of forcing frequency. Since it is obvious that a beam is a poor representation of a true orthotropic or composite structure due to the special orientation of fibers with the axis of a beam, it was decided to use a plate for a more realistic analysis of a composite structure. Also, a plate complies with the continuous system of a structure and eliminates the assumptions made during the log-decrement analysis of a beam in considering it as a single degree of freedom system.

2. THEORY

2.1 *Formulation of the problem*

For the set of coordinates considered (Fig. 1) the equation of motion for a specially orthotropic simply supported plate is given by[5]

$$\rho h\frac{\partial^2 w}{\partial t^2}+C\frac{\partial w}{\partial t}+D_x\frac{\partial^4 w}{\partial x^4}+D_y\frac{\partial^4 w}{\partial y^4}+H\frac{\partial^4 w}{\partial x^2\partial y^2}=q(x,y,t)$$

$$w=\frac{\partial^2 w}{\partial x^2}=\frac{\partial^2 w}{\partial y^2}=0,\quad \text{at } \begin{array}{l} x=0,a \\ y=0,b \end{array} \tag{2.1}$$

where

$$H=D_x\nu_{yx}+D_y\nu_{xy}+4D_{xy}$$

$$D_x=\frac{E_xh^3}{12\mu}$$

$$D_y=\frac{E_yh^3}{12\mu}$$

$$D_{xy}=\frac{G_{xy}h^3}{12}$$

$$\mu=(1-\nu_{yx}\nu_{xy}).$$

ρ, ν being the mass density and Poisson ratio, C the damping coefficient per unit area, and q the load per unit area acting perpendicular to the plate.

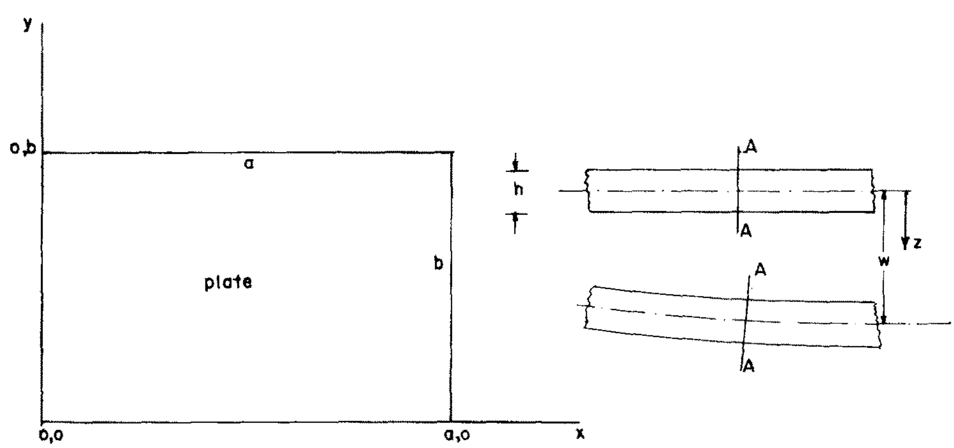

Fig. 1. Coordinate system used in analysis.

2.2 *Method of solution*

The equation of motion for a freely vibrating plate is

$$D_x \frac{\partial^4 w}{\partial x^4} + H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + \rho h \frac{\partial^2 w}{\partial t^2} + C \frac{\partial w}{\partial t} = 0. \tag{2.2}$$

The solution for the above equation can be written in the form

$$w(x, y, t) = \phi(x, y) e^{-\beta t} \cos \omega_{mn} t, \tag{2.3}$$

where ϕ represents the amplitude, β the damping parameter and ω_{mn} the damped circular frequency.

For eqn (2.3) to satisfy (2.2) at all time the damping parameter should be

$$\beta = \frac{C}{2\rho h}. \tag{2.4}$$

Equation (2.2) now reduces to

$$D_x \frac{\partial^4 \phi}{\partial x^4} + H \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 \phi}{\partial y^4} - \phi \lambda^4 = 0 \tag{2.5}$$

where

$$\lambda^4 = \rho h (\omega_{mm}^2 + \beta^2).$$

Using Levy's solution [6]

$$\phi_{mn} = \psi_m(x) \sin \frac{n\pi y}{b} \tag{2.6}$$

with the boundary condition

$$\psi_m(0) = \psi_m(a) = \left.\frac{\partial^2 \psi_m}{\partial x^2}\right|_{x=0} = \left.\frac{\partial^2 \psi_m}{\partial x^2}\right|_{x=a} = 0 \tag{2.7}$$

the solution to eqn (2.5) is given by

$$\psi_m(x) = A_m \sin \frac{m\pi x}{a}; \quad m\text{—an integer} \tag{2.8}$$

where A_m is an arbitrary constant. Now the natural frequency is obtained as

$$\begin{aligned} \omega_{0mn} &= (\omega_{mn}^2 + \beta^2)^{1/2} \\ &= \left\{ \frac{\pi^4}{\rho h} \left[D_x \frac{m^4}{a^4} + H \frac{m^2 n^2}{a^2 b^2} + D_y \frac{n^4}{b^4} \right] \right\}^{1/2}. \end{aligned} \tag{2.9}$$

This equation is the same as the one obtained by Hearmon [7] using the Rayleigh method.

Since it is easier to measure experimentally the phase angle between the plate vibration response and the applied forcing under steady state vibration of an externally excited plate, a load of the type

$$q(x, y, t) = P_0 \cos \Omega t \times \delta(x - a/2) \times \delta(y - b/2) \tag{2.10}$$

where Ω the forcing frequency has been considered. (Also such a point load can be applied practically at the center of the plate using an electro-magnetic shaker.)

To obtain the steady state solution of eqn (2.1), we assume a solution of the form

$$w(x, y, t) = \sum_m \sum_n \phi_{mn}(x, y) f_{mn}(t) \tag{2.11}$$

where ϕ_{mn} is given by eqn (2.6). Substituting eqn (2.11) in eqn (2.1) and using the orthogonal property of eigen functions, eqn (2.1) reduces to

$$\rho h\frac{d^2 f_{mn}}{dt^2}+C\frac{df_{mn}}{dt}+\lambda^4 f_{mn}=Q_{mn}(t) \tag{2.12}$$

where

$$Q_{mn}(t)=\frac{4}{ab}\int_0^a\int_0^b q(x,y,t)\phi_{mn}(x,y)\,dx\,dy.$$

Using eqn (2.10), Q_{mn} can be written as

$$Q_{mn}(t)=\frac{4}{ab}P_0\psi_m(a/2)\sin\frac{n\pi}{2}\cos\Omega t.$$

The homogeneous solution of eqn (2.12) is given by

$$f_{mn1}(t)=e^{-\beta t}(A_{mn}\cos\omega_{mn}t+B_{mn}\sin\omega_{mn}t) \tag{2.13}$$

where A_{mn} and B_{mn} are constants.

To determine the steady state solution of eqn (2.12), it is written in the form

$$\frac{d^2 f_{mn2}}{dt^2}+2\beta\frac{df_{mn2}}{dt}+(\omega_{mn}^2+\beta^2)f_{mn2}=\frac{4}{ab\rho h}\psi_m(a/2)\sin\frac{n\pi}{2}\cos\Omega t. \tag{2.14}$$

Letting

$$f_{mn2}=D_{mn}\cos\Omega t+C_{mn}\sin\Omega t \tag{2.15}$$

and substituting in the eqn (2.14), the constants D_{mn} and C_{mn} are obtained as

$$D_{mn}=\frac{4}{ab}\frac{P_0}{\rho h}\psi_m(a/2)\left(\sin\frac{n\pi}{2}\right)\left[\frac{\beta^2+\omega_{mn}^2-\Omega^2}{(\beta^2+\omega_{mn}^2-\Omega^2)^2+(2\beta\Omega)^2}\right] \tag{2.16}$$

and

$$C_{mn}=\frac{4}{ab}\frac{P_0}{\rho h}\psi_m(a/2)\left(\sin\frac{n\pi}{2}\right)\left[\frac{2\beta\Omega}{(\beta^2+\omega_{mn}^2-\Omega^2)^2+(2\beta\Omega)^2}\right]. \tag{2.17}$$

Hence the complete solution is

$$f_{mn}(t)=e^{-\beta t}[A_{mn}\cos\omega_{mn}t+B_{mn}\sin\omega_{mn}t]+[D_{mn}\cos\Omega t+C_{mn}\sin\Omega t]. \tag{2.18}$$

The initial conditions

$$f_{mn}(0)=0 \quad \text{and} \quad \frac{df_{mn}(0)}{dt}=0 \tag{2.19}$$

yield,

$$A_{mn}=-D_{mn} \tag{2.20}$$

and

$$B_{mn}=-\frac{1}{\omega_{mn}}[\beta D_{mn}+\Omega C_{mn}]. \tag{2.21}$$

The first part of the solution can be deleted as it tends to zero when time increases. From the second part of the solution we see that, ϕ, the phase angle between the applied force and the response is

$$\phi=\arctan\frac{\sum_m\sum_n C_{mn}}{\sum_m\sum_n D_{mn}}. \tag{2.22}$$

2.3 *Equation of energy loss in plates*

Equation (2.11) gives the solution to the equation of motion of a vibrating plate, excited centrally by a load of the type $P_0 \cos \Omega t \cdot \delta(x - a/2) \cdot \delta(y - b/2)$.

The energy loss per cycle due to viscous damping is given by

$$EL = \int_x \int_y \int_w C \frac{\partial w}{\partial t} \, dw \, dx \, dy. \tag{2.23}$$

Taking first few terms of eqn (2.11) and using eqn (2.23), the energy loss per cycle can then approximately be written for a square plate as

$$EL \simeq C\Omega \frac{a^2}{4} \pi (C_{11}^2 + D_{11}^2 + C_{33}^2 + D_{33}^2) \tag{2.24}$$

where C_{11}, C_{33}, D_{11}, D_{33}, are obtained by using eqns (2.16) and (2.17).

2.4 *Critical damping ratio*

Critical damping is defined as the damping necessary for no vibratory motion. This means that for critical damping the natural frequency of vibration is zero. For this case eqn (2.9) gives

$$\beta_{\text{critical}} = \omega_{0mn}. \tag{2.25}$$

Using eqn (2.4), the critical damping coefficient is obtained as

$$C_c = 2\rho h \omega_{0mn}. \tag{2.26}$$

Defining ζ the critical damping ratio as

$$\zeta = \beta / \beta_{\text{critical}}. \tag{2.27}$$

where β_{critical} being evaluated at $m = n = 1$; eqn (2.27) can be written as

$$\zeta = \beta / \omega_{011}. \tag{2.28}$$

3. EXPERIMENTAL PROCEDURE

A schematic diagram of the experimental setup is shown in Fig. 2. the test specimen (square plate) was excited at its center using an electromagnetic shaker. The phase lag between the force and the vibration response of the plate was obtained from the Lissajous diagram displayed on the

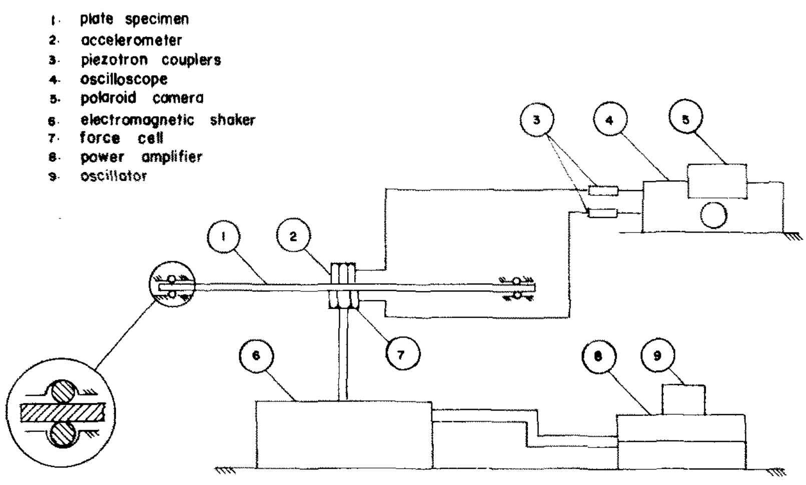

Fig. 2. Schematic diagram of experimental apparatus.

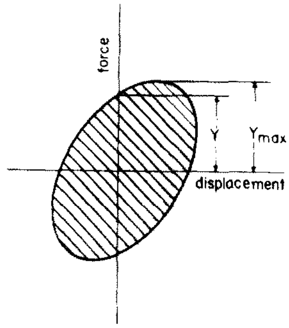

Fig. 3. Phase lag between applied force and resulting displacement at the plate midpoint.

oscilloscope screen, and using the expression from Fig. 3

$$\phi = \arc \sin\left(\frac{y}{y_{max}}\right) \tag{3.1}$$

the damping parameter was then calculated by means of eqn (2.22).

Also, the area enclosed by the Lissajous diagram was measured to determine experimentally the energy loss per cycle and compared with the theoretical energy loss predicted by using eqn (2.24). The above procedure was repeated for various vibration excitation frequencies.

Typical handbook property values were used for the steel and aluminum plates while the following mechanical properties were determined experimentally for calculating the natural frequency, damping and energy loss of the aluminum/graphite-fiber composite material:

Young's Modulus (Along the fiber direction)	$E_1 = 13.8 \times 10^6$ psi
Young's Modulus (Across the fiber direction)	$E_2 = 6.09 \times 10^6$ psi
Poison's Ratio	$\nu_{12} = 0.350$
Poisson's Ratio	$\nu_{21} = 0.159$
Rigidity Modulus	$G_{12} = 5.7 \times 10^6$ psi
Volume of Graphite Fibers	14%
Mass Density	$\rho = 2.85 \times 10^{-4}$ lb sec^2/in^4.

4. DISCUSSION AND CONCLUSION

Comparing the results of Refs. [8, 9] one can conclude that the determination of damping by the use of the log decrement method (single degree of freedom system) is largely influenced by the size of the beam, the support conditions, and the method used. Hence it is not always feasible to use the log decrement technique to determine reliable values for the equivalent viscous damping coefficient of a material. Since, for orthotropic or composite materials, such as aluminum graphite, the mechanical properties vary considerably in mutually perpendicular directions, the log decrement method utilizing a beam type element is not practical for damping measurements since the beam size does not permit a true representation of the actual material. Hence it is more appropriate to use a plate element (continuous system) to determine the damping coefficient of composite materials since the true character of the material can be represented.

From Figs. 4 and 5 one can conclude that for small amplitudes of transverse vibration, consistent with the bending analysis of plates, the phase angle and hence the damping parameter is primarily dependent on frequency and only very slightly dependent on the vibration amplitude.

It is evident from Fig. 6 that the critical damping ratio for the steel and aluminum samples varied somewhat with the ratio of the excitation frequency to the fundamental natural frequency. The damping data for the steel and aluminum plates of various thicknesses appear to give very consistent and uniform results. This tends to indicate that the critical damping ratio, and not the damping coefficient, may be treated as a physical material property.

The aluminum graphite damping results as also shown in the same figure indicate that the

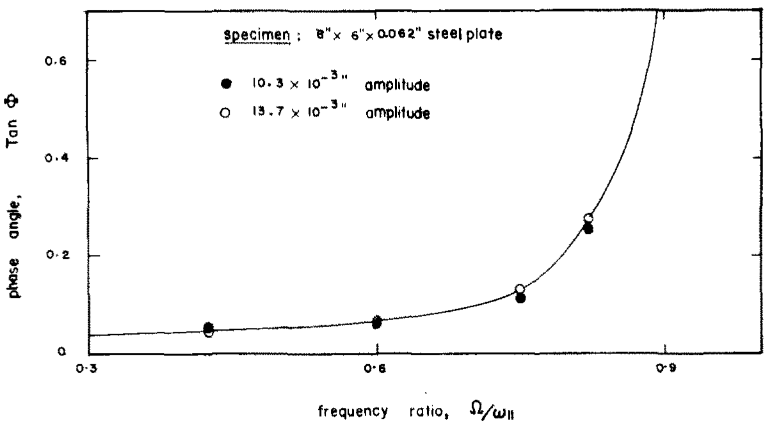

Fig. 4. Phase angle between applied force and resulting center displacement for various excitation frequencies at a constant vibration amplitude.

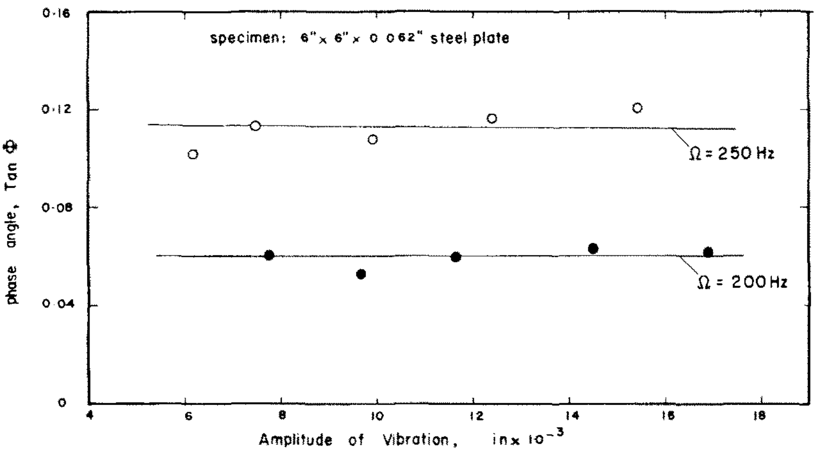

Fig. 5. Phase angle between applied force and center displacement at constant forcing frequency.

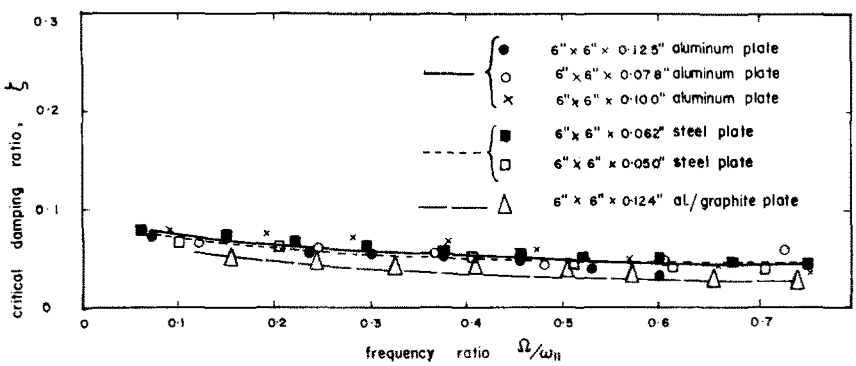

Fig. 6. Critical damping ratio variation with forcing frequency for various thickness steel, aluminum and aluminum-graphite composite plates.

critical damping ratio is considerably smaller than for the steel or aluminum. This damping ratio also decreases slightly with vibration frequency.

Figures 7 and 8 show the comparison between the theoretical and experimentally measured (area of the Lissajous diagram) energy loss per cycle of loading. The theoretical values have been calculated using the damping coefficients determined above by the continuous system analysis.

Consistent with the critical damping ratio data the energy loss per cycle for the aluminum-graphite plate sample was considerably less than for aluminum samples. This suggests that it may be possible to utilize composite materials for specific design applications where

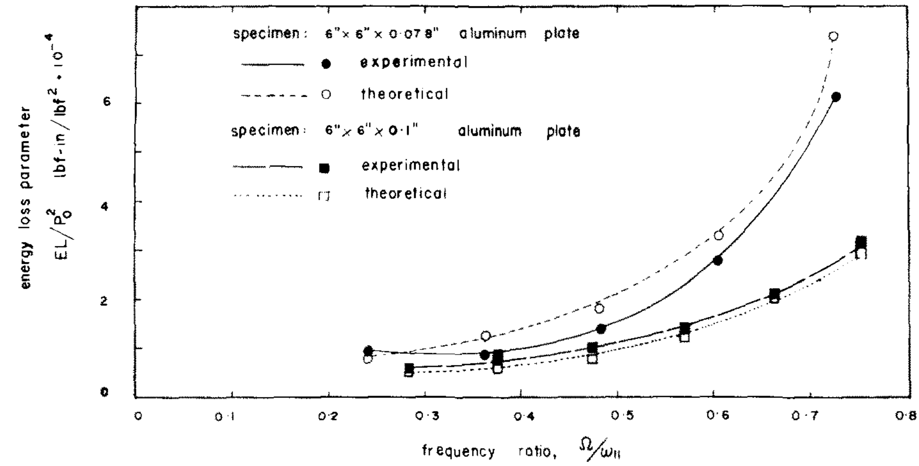

Fig. 7. Comparison of theoretical and experimental energy loss due to damping in aluminum plates.

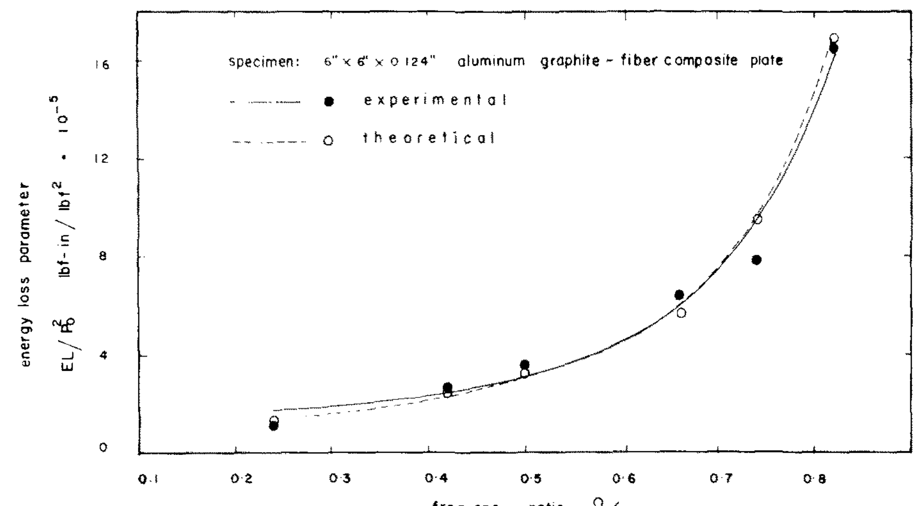

Fig. 8. Comparison of theoretical and experimental energy loss due to damping in the aluminum-graphite composite plate.

stiffness, strength and damping properties are desired. Present design considerations have been limited to the utilization of composites for their structural strength and stiffness characteristics. The results presented herein indicate that it is also possible to design a material for its dynamic damping characteristics.

The above results appear satisfactory considering any experimental errors involved and the theoretical approximations, and also show that the determination of damping coefficient using continuous system method is more realistic than that obtained by the single degree of freedom system (log decrement) analysis. By obtaining such data for different materials one can compare their effective damping at a particular or a range of frequencies. Also, it can be concluded that the viscous damping criteria can be effectively used for the analysis of composite materials which include damping.

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